CALCULATING FLUID FRICTION WITH FORMULAS

HEAD

FRICTION LOSS (Darcy-Weisbach)

\[
\frac{H_f}{L} \left( \frac{\text{ft of fluid}}{100/\text{ft of pipe}} \right) = \frac{1200 \times f}{D \text{ (in)}} \times \frac{(V C P h/2)}{2 \times g (\delta h/2)}
\]

VELOCITY

\[
v \ (ft/\text{s}) = 0.4085 \times \phi \ (gpm)
\]

REYNOLDS NUMBER

\[
Re = 7745.8 \times \frac{V (ft/\text{s}) \times D (\text{in})}{V (CST)}
\]

COLEBROOK EQUATION - FRICTION FACTOR \( f \)

\[
\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \frac{e (ft) + 2.5}{3.7 \times D (ft)} + \frac{Re \times \sqrt{f}}{Re \times \sqrt{f}} \right]
\]

LAMINAR FLOW EQUATION - FRICTION FACTOR \( f \)

\[
\frac{f}{Re} = \frac{64}{Re}
\]

\( H_f \): FRICTION LOSS
\( L \): PIPE LENGTH
\( f \): FRICTION PARAMETER
\( D \): INTERNAL PIPE DIA.
\( V \): VELOCITY
\( g \): GRAVITATIONAL ACC. 32.17 ft/s²
\( Re \): REYNOLDS NUMBER
\( \phi \): ABSOLUTE VISCOSITY
\( e \): PIPE ROUGHNESS

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DARCY-WEBERACH

Friction Factor \( f \)

\[
f = \frac{H_f}{L} \cdot \frac{L}{D} \cdot \frac{v^2}{2g} \cdot \frac{2 \cdot \epsilon}{2 \cdot \epsilon}
\]

\[
H_f = 1200 \times \frac{f}{100} \times \frac{(v/2)^2}{D} \times \frac{1}{100 \frac{\text{ft}}{\text{sec}}} \times \frac{1}{\frac{D}{2} \text{ ft}}
\]

Example: \( f = 0.03 \quad \epsilon = 0.00015 \quad D = 0.464 \text{ in} \quad \epsilon = 0.00015 \times 12 = 0.0038 \quad g = 32.17 \frac{\text{ft}}{\text{sec}^2} \quad D = \frac{0.464}{0.464}
\]

\[
V = \frac{0.4035 \times 1}{0.464} = 11.32
\]

\[
H_f = 1200 \times 0.03 \times \frac{11.38^2}{0.464 \times 2 \times 32.17} = 158
\]
**REYNOLDS NUMBER**

\[
Re = 7745.8 \times \frac{V(\text{m/s}) \times D(\text{m})}{\nu (\text{cSt})}
\]

**EXAMPLE:**

\[V = 11.38 \, \text{m/s}, \quad D = 0.464 \, \text{m}, \quad \nu = 1 \, \text{cSt}
\]

For water:

\[Re = 7745.8 \times \frac{11.38 \times 0.464}{1} = 4.09 \times 10^4
\]
The diagram illustrates the relationship between tangential stress, rate of shear, and viscosity in fluid dynamics. The equation for tangential stress is given by:

\[ T = \frac{1}{\mu} \times \frac{dv}{dy} \]

where:
- \( T \) is the tangential stress,
- \( \mu \) is the viscosity,
- \( \frac{dv}{dy} \) is the rate of shear (velocity gradient).
FRICTION FACTOR \( \frac{1}{f} \)

**COLE BROOK**

\[
\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{E(\alpha)}{3.7 \times D(\alpha)} + \frac{2.51}{Re \times (\alpha)} \right)
\]

**SWAMEE - JAIN**

\[
f = \frac{0.25}{\left( \log_{10} \left( \frac{E(\alpha)}{3.7 \times D(\alpha)} + \frac{5.74}{Re^{0.9}} \right) \right)^2}
\]

**EXAMPLE:** \( \varepsilon / D = 0.0038 \quad Re = 40,900 \)

\[
f = \frac{0.25}{\left( \log_{10} \left( \frac{0.0038}{3.7} + \frac{5.74}{40,900^{0.9}} \right) \right)^2} = 0.031
\]
LAMINAR FLOW

\[ f = \frac{64}{\text{Re}} \]

FLOW CORRESPONDS TO AVERAGE VELOCITY
Friction Factors for Commercial Pipe
(for Darcy-Weisbach formula, page 3-3)

Relative Roughness $= \frac{\varepsilon}{D}$

$\frac{\varepsilon}{D} = 0.0039$

$Re = 40000$

$Lamnlar Zone$

$Turbulent Zone$

$Complete Turbulence, Rough Pipes$

$\eta - Reynolds Number$
## PIPE ROUGHNESS VALUES

<table>
<thead>
<tr>
<th>Material</th>
<th>Absolute roughness (in x 10^-3)</th>
<th>Absolute roughness (micron or m x 10^-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveted steel</td>
<td>36-360</td>
<td>915-9150</td>
</tr>
<tr>
<td>Concrete</td>
<td>12-120</td>
<td>305-3050</td>
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<tr>
<td>Ductile iron</td>
<td>102</td>
<td>2591</td>
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<tr>
<td>Wood stave</td>
<td>3.6-7.2</td>
<td>91-183</td>
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<tr>
<td>Galvanized iron</td>
<td>6</td>
<td>152</td>
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<tr>
<td>Cast iron - asphalt dipped</td>
<td>4.8</td>
<td>122</td>
</tr>
<tr>
<td>Cast iron uncoated</td>
<td>10</td>
<td>254</td>
</tr>
<tr>
<td>Carbon steel or wrought iron</td>
<td>1.8</td>
<td>45</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>1.8</td>
<td>45</td>
</tr>
<tr>
<td>Fiberglass</td>
<td>0.2</td>
<td>5</td>
</tr>
<tr>
<td>Drawn tubing - glass, brass, plastic</td>
<td>0.06</td>
<td>1.5</td>
</tr>
<tr>
<td>Copper</td>
<td>0.06</td>
<td>1.5</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.06</td>
<td>1.5</td>
</tr>
<tr>
<td>PVC</td>
<td>0.06</td>
<td>1.5</td>
</tr>
<tr>
<td>Red brass</td>
<td>0.06</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Sources:
1. Cameron hydraulic data Book
2. Enginereed Software's PIPE-FLO software [www.engineered-software.com](http://www.engineered-software.com)
3. Fiberglass Pipe Handbook, SPI Composites Institute
Viscosity

\[ \nu (cP) = \frac{\mu (cP)}{SG} \]

\( V \): KINEMATIC VISCOITY
\( \mu \): DYNAMIC VISCOITY
\( SG \): SPECIFIC GRAVITY

\[ SG = \frac{\rho_F}{\rho_W} \]

\( \rho_F \): DENSITY OF FLUID
\( \rho_W \): DENSITY OF WATER AT STANDARD CONDITIONS

\( \rho_W = 62.4 \text{ lbm/ft}^3 \)
The Newton-Raphson iteration technique applied to the Colebrook equation

Since the value for \( f \) in the Colebrook equation cannot be explicitly extracted from the equation, a numerical method is required to find the solution. Like all numerical methods, we first assume a value for \( f \), and then, in successive calculations, bring the original assumption closer to the true value. Depending on the technique used, this can be a long or slow process. The Newton-Raphson method has the advantage of converging very rapidly to a precise solution. Normally only two or three iterations are required.

The Colebrook equation is:

\[
\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{2.51}{Re \sqrt{f}} \right)
\]

The technique can be summarized as follows:

1. Re-write the Colebrook equation as:

\[
F = \frac{1}{\sqrt{f}} + 2 \log_{10} \left( \frac{\varepsilon}{3.7D} + \frac{2.51}{Re \sqrt{f}} \right) = 0
\]

2. Take the derivative of the function \( F \) with respect to \( f \):

\[
\frac{dF}{df} = -\frac{1}{2} f^{-3/2} \left( 1 + \frac{2 \times 2.51}{\log_{10} 10 \times \left( \frac{\varepsilon}{3.7D} + \frac{2.51}{Re \sqrt{f}} \right) Re} \right)
\]

3. Give a trial value to \( f \). The function \( F \) will have a residue (a non-zero value). This residue \((RES)\) will tend towards zero very rapidly if we use the derivative of \( F \) in the calculation of the residue.

\[
f_n = f_{n-1} - RES \quad \text{with} \quad RES = \frac{F}{\frac{dF}{df}}
\]

For \( n = 0 \) assume a value for \( f_0 \), calculate \( RES \) and then \( f_n \), repeat the process until \( RES \) is sufficiently small (for example \( RES < 1 \times 10^{-6} \)).

The Newton-Raphson iteration technique is a method that converges very rapidly to a solution. You need to provide a seed value for \( f_{n-1} \) to start the iteration and an acceptable error \( RES \) which you can make very small.