CALCULATING FLUID FRICTION WITH FORMULAS

FRICTION HEAD LOSS (DARCY-WEISBACH)

$$\frac{H_F}{L}\left(\frac{ft\ of\ fluid}{100\ ft\ of\ pipe}\right) = \frac{1200 \times f}{D(in)} \times \frac{(V(ft/s))^2}{2 \times g\,(ft/s^2)}$$

VELOCITY

$$V(ft/s) = \frac{0.4085 \times Q\,(gpm)}{(D(in))^2}$$

REYNOLDS NUMBER

$$Re = 7745.8 \times \frac{V(ft/s) \times D(in)}{\nu\,(cSt)}$$

COLEBROOK EQUATION - FRICTION FACTOR f

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left[\frac{\epsilon\,(ft)}{3.7 \times D(ft)} + \frac{2.51}{Re \times \sqrt{f}}\right]$$

LAMINAR FLOW EQUATION - FRICTION FACTOR

$$f = \frac{64}{Re}$$

H_F: FRICTION LOSS
L: PIPE LENGTH
f: FRICTION PARAMETER
D: INTERNAL PIPE DIA.
V = VELOCITY
g = GRAVITATIONAL ACC. 32.17 ft/s^2
Re = REYNOLDS NUMBER
ν = ABSOLUTE VISCOSITY
ϵ = PIPE ROUGHNESS

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DARCY-WEISBACH

FRICTION FACTOR f

$$f = \frac{\frac{H_F}{L}}{\left(\frac{L}{D}\right) \times \frac{v^2}{2g}}$$

HEAD LOSS

VELOCITY HEAD

2 CHARACTERISTIC LENGTHS

$$\frac{H_F}{L}\left(\frac{\text{ft of fluid}}{100 \text{ ft pipe}}\right) = \frac{1200 \times f}{D\,(in)} \times \frac{(v\,(ft/s))^2}{2 \times g}$$

EXAMPLE: $f = 0.03$ FROM MOODY DIAGRAM $\quad \epsilon = 0.00015\ ft \quad D = 0.464\ in \quad g = 32.17\ ft/s^2 \quad \frac{\epsilon}{D} = \frac{0.00015 \times 12}{0.464} = 0.0038$

$$V = \frac{0.4085 \times 6}{0.464^2} = 11.38$$

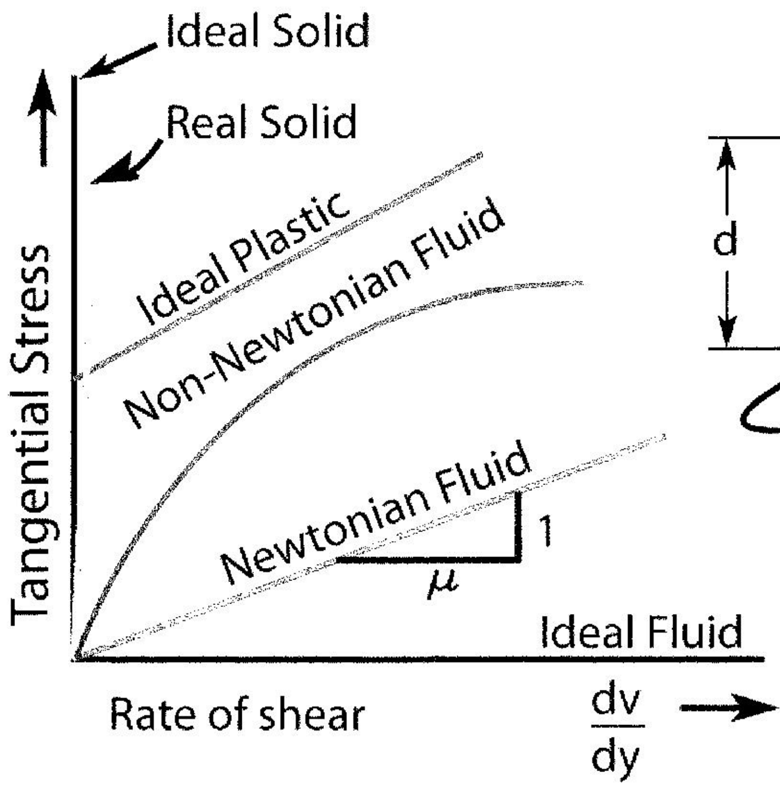
$$\frac{H_F}{L} = \frac{1200 \times 0.03}{0.464} \times \frac{11.38^2}{2 \times 32.17} = 158$$

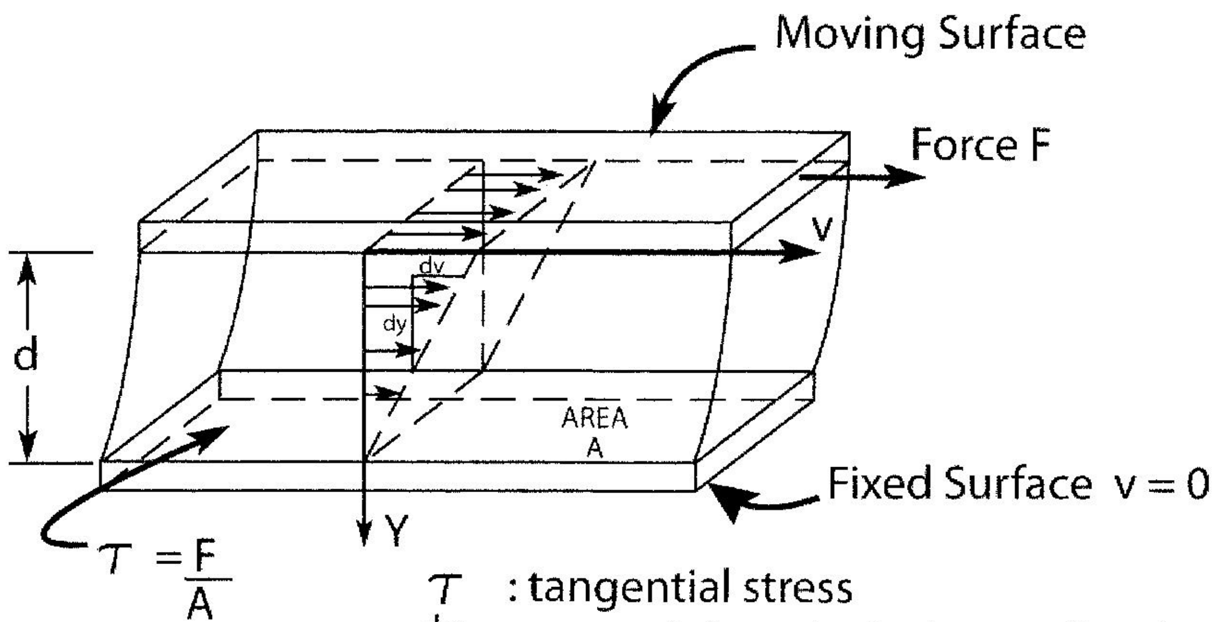
REYNOLDS NUMBER

$$R_e = 7745.8 \times \frac{V(ft/s) \times D(in)}{\nu \ (cSt)}$$

EXAMPLE : $V = 11.38$ $D = 0.464$ in $\nu = 1$ cSt FOR WATER

$$R_e = 7745.8 \times \frac{11.38 \times 0.464}{1} = 4.09 \times 10^4$$





τ : tangential stress

$\frac{dv}{dy}$: rate of shear (velocity gradient)

μ : viscosity

$$\tau = \frac{1}{\mu} \times \frac{dv}{dy}$$

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FRICTION FACTOR f

COLEBROOK

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{\epsilon(ft)}{3.7 \times D(ft)} + \frac{2.51}{Re \times \sqrt{f}}\right)$$

SWAMEE - JAIN

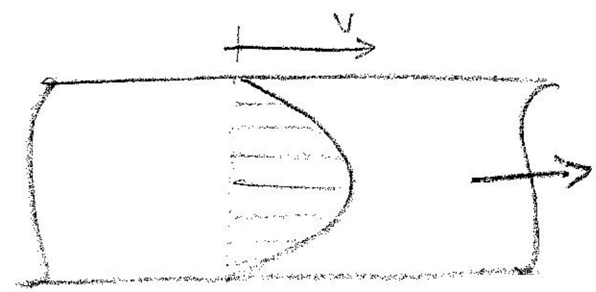
$$f = \frac{0.25}{\left(\log_{10}\left(\frac{\epsilon(ft)}{3.7 \times D(ft)} + \frac{5.74}{Re^{0.9}}\right)\right)^2}$$

EXAMPLE: $\epsilon/D = 0.0038$ $\quad Re = 40900$

$$f = \frac{0.25}{\left(\log_{10}\left(\frac{0.0038}{3.7} + \frac{5.74}{40900^{0.9}}\right)\right)^2} = 0.031$$

LAMINAR FLOW

$$f = \frac{64}{Re}$$

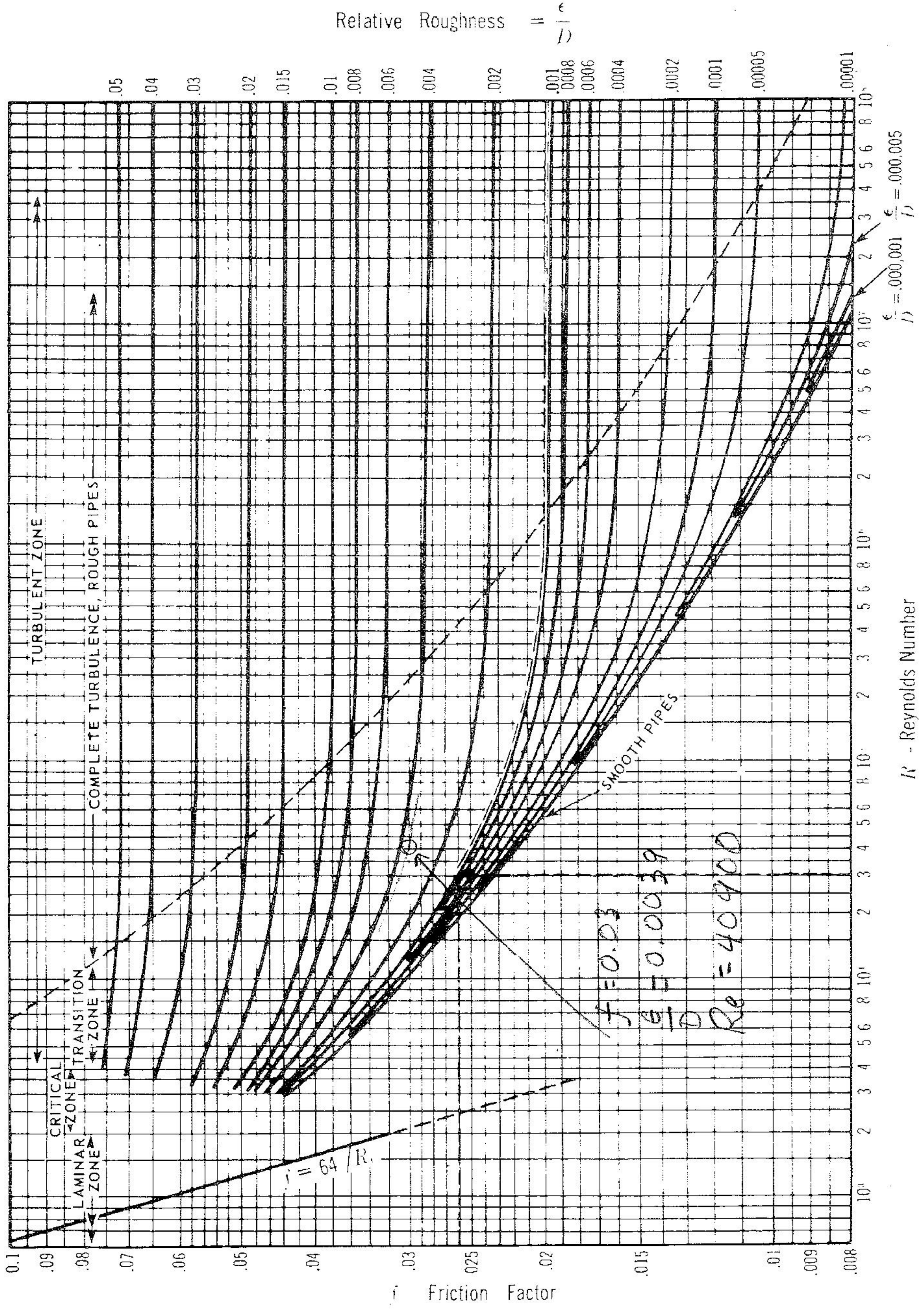


FLOW CORRESPONDS
TO AVERAGE VELOCITY

7.

Friction Factors for Commercial Pipe

(for Darcy-Weisbach formula, page 3-3)



PIPE ROUGHNESS VALUES

Pipe absolute roughness values (RMS)		
Material	Absolute roughness (in x 10^{-3})	Absolute roughness (micron or m x 10^{-6})
Riveted steel[1]	36-360	915-9150
Concrete[1]	12-120	305-3050
Ductile iron[2]	102	2591
Wood stave[1]	3.6-7.2	91-183
Galvanized iron[1]	6	152
Cast iron – asphalt dipped[1]	4.8	122
Cast iron uncoated[1]	10	254
Carbon steel or wrought iron[1]	1.8	45
Stainless steel[1]	1.8	45
Fiberglass[3]	0.2	5
Drawn tubing – glass, brass, plastic[1]	0.06	1.5
Copper[2]	0.06	1.5
Aluminium[2]	0.06	1.5
PVC[2]	0.06	1.5
Red brass[2]	0.06	1.5

Sources :
1. Cameron hydraulic data Book
2. Engineered Software's PIPE-FLO software www.engineered-software.com
3. Fiberglass Pipe Handbook, SPI Composites Institute

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VISCOSITY

$$\nu\,(cSt) = \frac{\mu\,(cP)}{SG}$$

ν: KINEMATIC VISCOSITY
μ: DYNAMIC VISCOSITY
SG: SPECIFIC GRAVITY

$$SG = \frac{\rho_F}{\rho_W}$$

ρ_F: DENSITY OF FLUID

ρ_W: DENSITY OF WATER AT STANDARD CONDITIONS 62.34 lbm/ft^3

The Newton-Raphson iteration technique applied to the Colebrook equation

Since the value for f in the Colebrook equation cannot be explicitly extracted from the equation, a numerical method is required to find the solution. Like all numerical methods, we first assume a value for f, and then, in successive calculations, bring the original assumption closer to the true value. Depending on the technique used, this can be a long or slow process. The Newton-Raphson method has the advantage of converging very rapidly to a precise solution. Normally only two or three iterations are required.

The Colebrook equation is:

$$\frac{1}{\sqrt{f}} = -2\,\log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\sqrt{f}}\right)$$

The technique can be summarized as follows:

1. Re-write the Colebrook equation as:

$$F = \frac{1}{\sqrt{f}} + 2\,\log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\sqrt{f}}\right) = 0$$

2. Take the derivative of the function F with respect to f:

$$\frac{dF}{df} = -\frac{1}{2}f^{-3/2}\left(1 + \frac{2\times 2.51}{\log_e 10\times\left(\dfrac{\varepsilon}{3.7D} + \dfrac{2.51}{\mathrm{Re}\sqrt{f}}\right)\mathrm{Re}}\right)$$

3. Give a trial value to f. The function F will have a residue (a non-zero value). This residue (RES) will tend towards zero very rapidly if we use the derivative of F in the calculation of the residue.

$$f_n = f_{n-1} - RES \quad with \quad RES = \frac{F}{\dfrac{dF}{df}}$$

For $n = 0$ assume a value for f_0, calculate RES and then f_1, repeat the process until RES is sufficiently small (for example $RES < 1 \times 10^{-6}$).

The Newton-Raphson iteration technique is a method that converges very rapidly to a solution. You need to provide a seed value for fn-1 to start the iteration and an acceptable error RES which you can make very small.