

**Energy and power determination for the Pelton turbine water jet**  
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The energy of an object of mass  $m$  with a velocity  $v$  is its kinetic energy and is expressed as:

$$E = \frac{1}{2} \times m \times v^2$$

Since power is energy per unit time then:

$$P = \frac{1}{2} \times m/t \times v^2$$

The quantity  $m/t$  is the mass flow rate and can be expressed as the volumetric flow rate  $Q$  times the density  $\rho$ , therefore the power is:

$$P = \frac{1}{2} \times Q \times \rho \times v^2 \quad \mathbf{(1)}$$

This is the power available from an open water jet; the only energy available is its kinetic energy.

Another way to express power is to use head.

Head is an energy term or more precisely energy per unit weight displaced and is frequently used with fluid system because it simplifies certain relationships. The unit of head in the Imperial system is feet which is a simplification of  $\text{lbf-ft/lbf} = \text{ft}$ , or energy per unit weight.

The other power equation that includes the head, actually this is the net head at the water jet as it leaves the nozzle is:

$$P = \gamma \times h \times Q \quad \mathbf{(2)}$$

Where  $\gamma$  is the specific weight of the liquid and is equal to  $\rho \times g$  where  $g$  is the acceleration due to gravity and equal to  $32.17 \text{ ft/s}^2$  in the Imperial system. To understand how we arrive at the net head  $h$ , it is necessary to do an energy balance of the system. The energy between points 1 (the level of the stream surface) and point 2 (the level of the water jet nozzle) must be conserved and all energy loss due to friction must be accounted for.

$$\frac{p_1}{\rho} + mgz_1 + \frac{1}{2}mv_1^2 = \frac{p_F}{\rho} + \frac{p_2}{\rho} + mgz_2 + \frac{1}{2}mv_2^2 \quad \mathbf{(3)}$$

Equation (3) expresses the energy available at point 1 and the energy available at point 2 plus the energy lost through friction  $p_F/\rho g$ . The other terms are the pressure energy  $p_1/\rho g$  and the same at point 2. If you are wondering how come  $p_1/\rho g$  is energy terms just replace the variables with their units, making sure they are consistent, and you will come up with an energy term. In our case and  $p_1$  and  $p_2$  are zero since there is no pressure at these points. Equation (3) becomes:

$$mgz_1 + \frac{1}{2}mv_1^2 = \frac{p_F}{\rho} + mgz_2 + \frac{1}{2}mv_2^2 \quad (4)$$

if we divide all the terms in equation (4) by  $mg$  we will get all head terms, remember heads is energy per unit weight or  $mg$ .

$$z_1 = H_F + z_2 + \frac{1}{2g}v_2^2 \quad (5)$$

In our case the level of the stream is assumed to be constant so that  $v_1 = 0$  and the term  $p_F/\rho mg$  is a head term and we call this the friction head loss  $H_F$ .

If we rearrange equation (5):

$$z_1 - z_2 - H_F = \frac{1}{2g}v_2^2 = h \quad (6)$$

We see that on one side we have the static head minus the energy loss which gives us the kinetic energy at point 2 which is equal to the net head that we call  $h$ . From equation (6) we see that  $hg = v^2/2$  and if we use this in equation (1) we get:

$$P = Q \times \rho \times h \times g$$

And since  $\gamma = \rho$  then

$$P = \gamma \times h \times Q$$

which is equation (2).