APPENDIX B

THE NEWTON-RAPHSON ITERATION TECHNIQUE APPLIED TO THE COLEBROOK EQUATION

APPENDIX B

THE NEWTON-RAPHSON ITERATION TECHNIQUE

Since the value for f in the Colebrook equation cannot be explicitly extracted from the equation, a numerical method is required to find the solution. Like all numerical methods, we first assume a value for f, and then, in successive calculations, bring the original assumption closer to the true value. Depending on the technique used, this can be a long or slow process. The Newton-Raphson method has the advantage of converging very rapidly to a precise solution. Normally only two or three iterations are required.

The Colebrook equation is:

$$\frac{1}{\sqrt{f}} = -2\ \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\sqrt{f}}\right)$$

The technique can be summarized as follows:

1. Re-write the Colebrook equation as:

$$F = \frac{1}{\sqrt{f}} + 2\ \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e\sqrt{f}}\right) = 0$$

2. Take the derivative of the function F with respect to f:

$$\frac{dF}{df} = -\frac{1}{2} f^{-3/2}\left(1 + \frac{2\times 2.51}{\log_e 10 \times \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)\mathrm{Re}}\right)$$

3. Give a trial value to f. The function F will have a residue (a non-zero value). This residue (RES) will tend towards zero very rapidly if we use the derivative of F in the calculation of the residue.

$$f_n = f_{n-1} - RES \quad with \quad RES = \frac{F}{\frac{dF}{df}}$$

For $n = 0$ assume a value for f_0, calculate RES and then f_1, repeat the process until RES is sufficiently small (for example $RES < 1 \times 10^{-6}$).